

RSA experience

Bob wants to **receive** secret messages

Choose 2 prime numbers p_1 and p_2

$$p_1 = 11$$

$$p_2 = 13$$

Compute public $n = p_1 * p_2$:

$$n = 143$$

Compute $\phi(n) = (p_1 - 1) * (p_2 - 1)$ and keep it private

$$\phi(n) = 10 * 12 = 120$$

Choose public encryption exponent e to be a random prime number less than ϕ that is also not a divisor of ϕ , but such that $k\phi(n) + 1$ is divisible by this number

Non-divisors of ϕ : 1, 2, 3, 4, 5, 6, **7**, 8, 9, ...

Chosen public $e = 7$

Post n and e for everyone to use: $n = 143$, $e = 7$

Compute decryption exponent $d = (k\phi(n) + 1) / e$ (must be an integer)

$$(1 * 120 + 1) / 7 = 17.3$$

$$(2 * 120 + 1) / 7 = 34.4$$

$$(3 * 120 + 1) / 7 = 51.6$$

$$(4 * 120 + 1) / 7 = 68.7$$

$$(5 * 120 + 1) / 7 = 85.9$$

$$(6 * 120 + 1) / 7 = \underline{\mathbf{103}}$$

$$d = (6 * 120 + 1) / 7 = \underline{\mathbf{103}}$$

$$d = 103$$

Alice wants to **send** secret messages

Get public values of n and e : $n = 143$, $e = 7$

Select a secret number to send to Bob (make it a small prime to be a coprime with n):

Alice wants to send number $x = 19$

Encrypt it using formula $x^e \bmod n$, and e and n provided by Bob.

She computes encrypted number $y = x^e \bmod n = 19^7 \bmod 143 = 46$

<https://www.mtholyoke.edu/courses/quenell/s2003/ma139/js/powermod.html>

Send $y = 46$

Bob receives secret messages

Receive y from Alice:

$y = 46$

Bob decrypts it using decryption exponent d :

$x = y^d \bmod n$

$x = 46^{103} \bmod 143 = 19$